SDMC Fermat, November 8, 2008
PROBLEMS

Note 1: *Unless otherwise stated, all answers must be exact, complete, and in simplest form. "Exact" means that there are no approximations, which means that where applicable, answers are left as fractions in terms of π and radicals. "Complete" means that all correct answers, and nothing else, must be part of your answer. "Simplified" includes things such as completely reducing all fractional quantities and expressing irrational quantities in simplest radical form. For example, $\sqrt{24}$ is not simplified, because the simplest radical form requires no perfect square factors under the radical: $\sqrt{24} = 2\sqrt{6}$. Answers such as $\frac{3}{\sqrt{2}}$ are not considered simplified, because it is inconvenient to have irrational numbers in the denominators: $\frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$ Answers such as $2(1+\sqrt{2})$ are not considered simplified either, because an operation remains that can be performed exactly: $2(1+\sqrt{2}) = 2+2\sqrt{2}$.*

Note 2: *To "cut a vertex" of a polygon or a polyhedron, draw a line or a plane that intersects all edges adjacent to that vertex somewhere between their endpoints (NOT AT the endpoints).*

1. What is the measure of a single angle of a regular polygon with 36 sides?
2. All angles of a regular polygon are acute. How many sides does it have?
3. A regular polygon is partially covered with a sheet of white paper as shown in Figure 1. The measures of the marked angles are 39° and 51°. How many sides does the polygon have?
4. Find the sum of shaded angles in a five-pointed star in Figure 2.
5. An *external angle* is an angle formed by one side of a polygon and a line extended from an adjacent side, as shown in Figure 3. Find the sum of external angles of a regular 96-gon.
6. All sides of the pentagon $ABCDE$ have the same length. However, it is not regular, because the angle measures are not all the same. Adjacent angles A and B measure 90° each. What is the measure of angle C?
7. For how many different positive integers n does a convex regular polygon with n sides have angles whose measures, in degrees, are integers?
8. A regular octagon is formed by cutting an isosceles right triangle from each of the corners of a square with sides of length 3. What is the length of each side of the octagon? (*Refer to the text on top of this page for the correct form of the answer*).
9. A convex polyhedron has 25 faces and 25 vertices. Find the number of edges of this polyhedron.
10. What is the name of a solid obtained by folding a pattern in Figure 4?
11. Name a convex polyhedron with 5 faces and 8 edges.
12. By cutting out one angle of a square table, how many angles are obtained?
13. In a cube with the edge length of 1, one vertex is chosen. Then the cube is cut by a plane *passing through all vertices adjacent to the chosen vertex*. Find the volume of the new solid containing the chosen vertex.
14. Give V, E, and F (the number of vertices, edges, and faces) for a polyhedron obtained by cutting out all vertices of a cube.

15. Give V, E, and F for a polyhedron obtained by cutting out the vertex of a rectangular pyramid that is opposite to the base.

16. Give V, E, and F for a polyhedron obtained by cutting out a single base vertex of a tetragonal pyramid.

17. Find the volume of an octahedron obtained by connecting the centers of faces of a cube with the edge length of 3.

18. Find the volume of a cube obtained by connecting the centers of faces of a regular octahedron with the edge length of 3. *Hint: draw the "top view" of the octahedron (the view along the line connecting two opposite vertices); sketch centers of the faces and the side of the cube connecting them.*

19. An ant desires to walk a surface of a cube, visiting each face at least once. Starting from the center of one of the faces, what is the minimum number of edges that the ant has to cross?

20. Same ant now wants to walk the surface of a tetrahedron, starting from the center of one of the faces and returning to the same point at the end. What is the minimum number of edges to be crossed?

21. On a piece of paper, ten points are connected by 16 line segments so that no two segments intersect. How many regions (not counting the exterior) are bounded by these segments?

22. On a piece of paper, 100 points are connected by 160 line segments so that no two segments intersect. How many regions (not counting the exterior) are bounded by these segments?

23. What is the maximal number of regions into which a plane can be divided when a square and a circle are drawn on it? Count the exterior this time.

24. Four regular tetrahedrons with the edge length of 1 are pushed in the corners of a regular tetrahedron with the edge length of 2. What is the shape of the empty space inside the big tetrahedron?

25. A regular octahedron and a regular tetrahedron have the same surface area. What is the answer when the edge length of the tetrahedron is divided by the edge length of the octahedron?

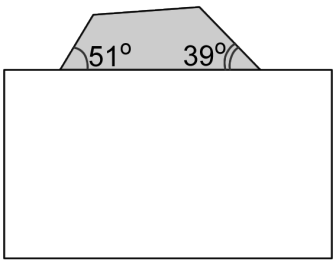

Figure 1.

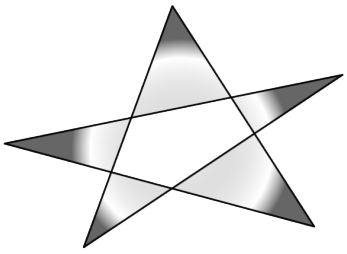
Figure 2.

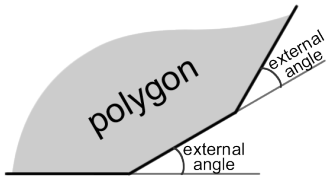

Figure 3.

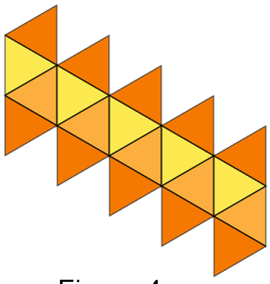
Figure 4.

Bonus Question: Find the ratio of the empty volume in problem #24 to the volume of one small tetrahedron. Combined with the *shape* of the empty space, it is a pretty neat result!